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# DEFORMATION OF THE SURFACE OF A CONDUCTING LIQUID UNDER THE ACTION OF A PULSE OF A STRONG FIELD

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*It has been shown that the development of the process of the so-called Frenkel–Tonks instability occurring in the case of application of an electric field to a plane surface of a conducting liquid is also determined by the size of the initial distortions of the surface.*

The spectrum of capillary waves on the charged surface of a conducting liquid is known [1, 2] to be written in the form

$$\omega(k) = \sqrt{\frac{\gamma}{\rho} k^3 - \frac{4\pi\sigma^2}{\rho} k^2} \; . \tag{1}$$

where $\sigma$ is related to the field strength $E$ near the liquid surface by the relation $\sigma = E/4\pi$. The deviation of the surface from equilibrium because of the wave motion is written as follows:

$$\xi = \xi_0 \exp(ikx - i\omega t) \; , \tag{2}$$

Equation (1) determines the occurrence and development of the well-known Frenkel–Tonks instability: in fairly strong fields, when the radicand becomes negative, the amplitudes of waves of the corresponding length build up with time. This brings up the questions: waves of what length oscillate most strongly and what does this depend on? Below we show that the development of the process is determined not only by the values of the field acting on the surface and the times of its action but also by the size of the initial distortions of the surface (i.e., of its deviations from equilibrium, for example, due to fluctuations).

Thus, there is a conducting liquid the deviation of whose surface from equilibrium at the initial instant of time can be represented in the form of the Gauss distribution

$$a(x) = a_0 \exp\left(-\frac{x^2}{x_0^2}\right). \tag{3}$$

The Fourier expansion of function (3) has the form [3]

$$a(x) = \int_{-\infty}^{\infty} dk a_k \exp(ikx) \; , \quad a_k = \frac{x_0 a}{2\sqrt{\pi}} \exp\left(-\frac{k^2 x_0^2}{4}\right). \tag{4}$$

The evolution of any initial deformation with time is determined as

$$a(x, t) = \int_{-\infty}^{\infty} dk a_k \exp(ikx - i\omega(k)\, t) \quad (t \geq 0) \; . \tag{5}$$

The electric field does not act on the surface until the zero instant of time begins; it switches on at the instant $t = 0$ and acts to the instant $t = \tau$; thereafter, it switches off. We consider the variant of a strong field where the inequality $4\pi\sigma^2 >> \gamma/x_0$ holds.

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In this case, according to (1), the expression for $\omega(k)$ is simplified:

$$\omega(k) = \pm i \sqrt{\frac{4\pi\sigma^2}{\rho}}\, k . \qquad (6)$$

To separate the solution building up in amplitude we select the root with a plus sign. The employment of expression (6) enables us to obtain a simple analytical form describing the deformation of the surface at the instant of time $t = \tau$ and at subsequent times. It is precisely the Fourier component of the amplitude $a_k$ at the instant of time $\tau$ that is equal to

$$a_k(\tau) = a_k \exp(\kappa k) , \qquad (7)$$

where

$$\kappa = \sqrt{\frac{4\pi\sigma^2}{\rho}}\, \tau . \qquad (8)$$

Substituting (7) into (5) (or into (4)), we find

$$a(x, \tau) = a \exp\left(-\frac{x^2}{x_0^2} + \frac{\kappa^2}{x_0^2}\right) \cos \frac{2x\kappa}{x_0^2} . \qquad (9)$$

The latter formula shows that the initial distortion grows in amplitude as a result of the action of a strong electric pulse: the amplitude increases by a factor of exp $(\kappa^2/x_0^2)$ at the point $x = 0$. Simultaneously, there develops a wave whose length, according to (9), is equal to

$$\lambda = \frac{\pi x_0^2}{\kappa} , \qquad (10)$$

i.e., the wave is the shorter, the stronger the field and the longer its action. The wavelength has a quadratic dependence on the scale of the initial inhomogeneity and is independent of the amplitude of the latter. The wave manifests itself in relief if its length is substantially smaller than the initial scale of inhomogeneity, i.e., if $\lambda << x_0$ or, in other words, $\kappa >> \pi x_0$, which means the limit of the strong electric pulse again.

Thus, the action of a pulsed electric field on a conducting surface is accompanied by an increase in the amplitude of the initial distortions of the surface (including fluctuations of various kind) with a simultaneous spatial modulation of these distortions, whose wavelength is the shorter, the larger the product of the field strength by the pulse duration ($E\tau$) and the smaller the initial inhomogeneity scale.

## NOTATION

$a_0$, initial amplitude of deviation, m; $k$, wave number of the surface wave, 1/m; $t$, time, sec; $x_0$, characteristic spatial scale of deviation, m; $x$, in any axial coordinate along the surface, m; $\gamma$, coefficient of surface tension of the liquid, N/m; $\rho$, density of the liquid, kg/m$^3$; $\sigma$, surface density of electric charge, C/m$^2$; $\omega$ and $\xi$, frequency and amplitude of the surface wave, 1/sec and m.

## REFERENCES


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